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Acceleration phenomenon in the synchronization of diffusively coupled oscillators

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Abstract

Synchronization of randomly coupled networks, with each node being a van der Pol oscillator subject to parametric excitation, is studied in the present paper. The effects of the network structure, the initial conditions and the intensity of Gaussian white noise on the synchronization performance of diffusively coupled oscillators are also investigated. It is found that unidirectionally coupled dynamical networks with proper parametric excitation can achieve synchronization, and it is interesting to reveal that stochastic excitation can even accelerate network synchronization under certain initial conditions and initial intensity of stochastic excitation.

PACS numbers: 05.45.Xt, 87.19.ln, 05.40.−a, 05.40.Ca

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Synchronization of oscillations has been an important subject of research since Huygens' observation about the synchrony between pendulum clocks [1]. It is now widely studied and applied in many scientific fields, such as physics, biology, electronic engineering, and meteorological, economics and even social sciences [3–7].

A complex network is a large set of interconnected nodes, in which a node is a fundamental unit which can have different meanings in different situations such as chemical substrates, microprocessors, computers, schools, companies, papers, webs, individual people, and so on, and the connections among them are repressed by links (or edges) [8–12]. Synchronization of complex networks of dynamical systems has received a great deal of attention particularly in the past decade. Several types of synchronization have been

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theoretically defined and experimentally observed in recent studies, such as completed synchronization, lag synchronization, phase synchronization and generalized synchronization [13–15]. Some results on network synchronization have been obtained in the literature. To mention just a few, Chen, Zhou and Liu proved that global synchronization of coupled DNNs can be achieved by a suitably designed coupling matrix and an inner linking matrix [16]. Goltsev *et al* developed a phenomenological theory of critical phenomena in networks with an arbitrary distribution of connectivity [17]. Cao and Wang studied some sufficient conditions for synchronization in an array of linearly coupled networks with time-varying delay based on the Lyapunov functional method and the matrix inequality technique [17]. Samoletov *et al* investigated the phenomenon of the spatiotemporal stochastic resonance in a chain of diffusively coupled bistable oscillators [18]. Chen and Zhou studied synchronization of uncertain generic complex networks [19]. Last but not least, Lu and Chen provided a new approach to network synchronization analysis [20]. Arenas *et al* presented an overview of the recent developments in synchronization of complex networks [21].

Noise is ubiquitous in both nature and manmade systems. It is usually regarded as random and persistent disturbance, which obscure or reduce the clarity of signals. In related studies, Lin and Chen showed that chaos synchronization could be achieved with probability 1 merely via white-noise-based coupling [22], and Lin and He showed some sufficient conditions for achieving complete synchronization between unidirectionally coupled Chua's circuits with stochastic perturbations [23]. Moreover, Nandi *et al* studied chemical reactions and genetic networks with stochastic perturbations [24], and Korniss studied synchronization of weighted uncorrelated scale-free networks in a noisy environment [25].

This paper continues and extends the above research endeavors to further investigate the synchronization of networks with parametric excitation by Gaussian white noise and the synchronization of randomly coupled networks. In this study, the main concern is the effects of the network structure, the initial conditions and the intensity of Gaussian white noise on the synchronization performance of networks with parametric excitations. The investigation is basically numerical, given the fact that there are no theoretical results available in the literature that can be used for analysis due to the complicated and difficult nature of the stochastic network setting.

The rest of the paper is organized as follows. In section 2, a unidirectionally randomly coupled dynamical network within noise perturbation is presented, where some preliminaries are also provided. In section 3, several numerical examples are analyzed and discussed, with simulation results demonstrated. Finally, section 4 concludes the paper.

2. The network model

Consider a dynamical network consisting of N linearly and diffusively coupled identical nodes, with each node being a van der Pol oscillator, subject to parametric excitation:

$$\begin{cases} \dot{x}_{i1} = x_{i2} + (\beta + \xi_2(t)) \displaystyle\sum_{j=1}^{N} c_{ij} a_{11} x_{j1} \\ \dot{x}_{i2} = -(\alpha_1 + \xi_1(t)) x_{i2} - \alpha_2 x_{i2}^3 - \omega^2 x_{i1} + (\beta + \xi_2(t)) \displaystyle\sum_{j=1}^{N} c_{ij} a_{22} x_{j2} \end{cases} \qquad i = 1, 2, \ldots, N, \tag{1}$$

where $(x_{i1}, x_{i2})^T \in R^2$ is the state vector of the ith node; a_{11} and a_{22} are the inner-displacement-coupling coefficient and inner-velocity-coupling coefficient, respectively; $C = (c_{ij})_{N\times N}$ is the

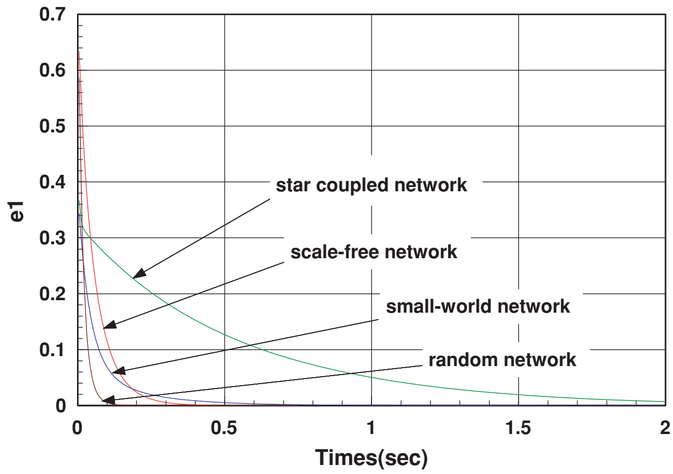

Figure 1. Synchronization displacement error in four types of networks subject only to parametric excitation: star-coupled network, randomly coupled network ($r = 0.05$), small-world network ($K = 6$, $r = 0.005$), scale-free network (the initial network is a randomly coupled network ($r = 0.1$, $N_0 = 20$, and every new node brings $m_0 = 10$ connections to the network). The time step and the total time are $\delta t = 0.005$, $T = 2.0$, respectively, and the initial conditions are randomly uniformly distributed, with $d_1 = 0.001$ and $d_2 = 0$.

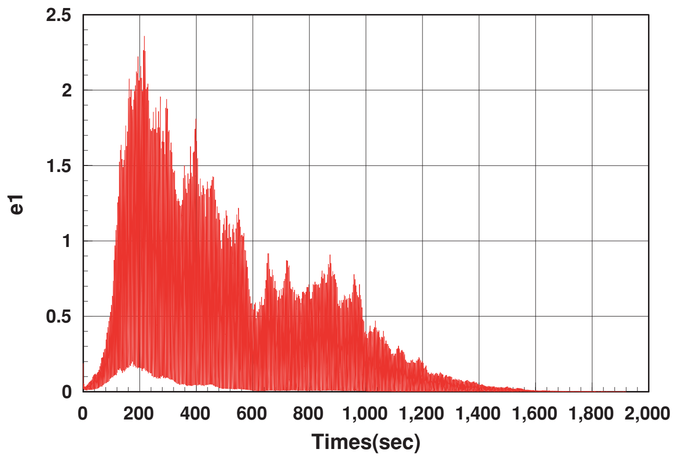

Figure 2. Synchronization displacement error in a nearest-neighbor coupled network ($K = 6$). The time step is $\delta t = 0.06$, and the initial conditions are randomly uniformly distributed, with $d_1 = 0.001$ and $d_2 = 0$.

coupling configuration matrix, which represents the topological structure of the network and is defined as follows: if there is no connection from node i to node j ($j \neq i$) then $c_{ij} = c_{ji} = 0$; otherwise, $c_{ij} = c_{ji} = 1$ ($j \neq i$); the diagonal elements of matrix C are defined by

$$c_{ii} = -\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij} \qquad i = 1, 2, \ldots, N, \tag{2}$$

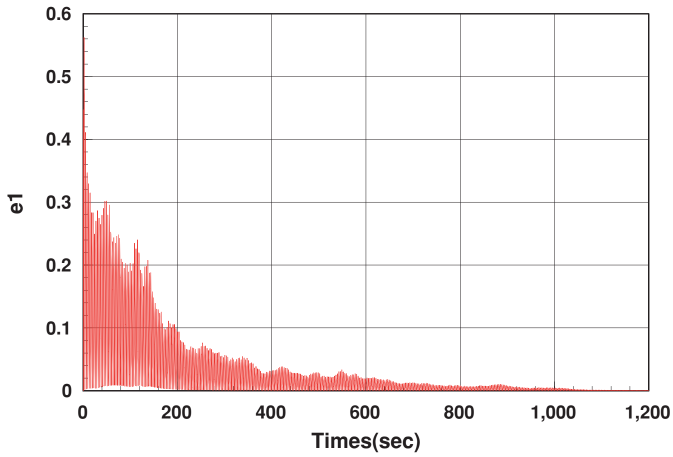

Figure 3. Synchronization displacement error in a nearest-neighbor coupled network ($K = 6$). The time step is $\delta t = 0.06$ and the initial conditions are $\left\{ {x_{i1}(0) \atop x_{i2}(0)} \right\} = \left\{ {1+2\times(i-N/2)/N \atop 1-2\times(i-N/2)/N} \right\}$, with $d_1 = 0.001$ and $d_2 = 0$.

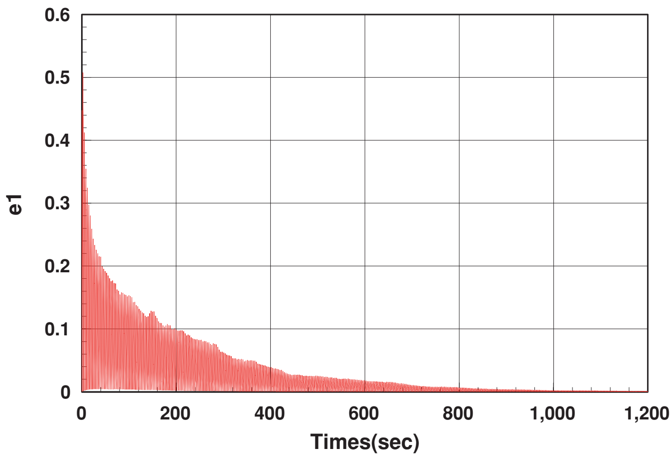

Figure 4. Synchronization displacement error in a nearest-neighbor coupled network ($K = 6$). The time step is $\delta t = 0.06$ and the initial conditions are $\left\{ {x_{i1}(0) \atop x_{i2}(0)} \right\} = \left\{ {1+2\times(i-N/2)/N \atop 1-2\times(i-N/2)/N} \right\}$, with $d_1 = 0.001$ and $d_2 = 0$.

β is the mean coupling strength of the network; $\xi_1(t)$ and $\xi_2(t)$ are independent Gaussian white noises with intensities d_1 and d_2, respectively.

Five types of networks, namely, star-coupled networks, randomly coupled networks, small-world networks, scale-free networks and nearest-neighbor coupled networks, are considered in this paper. The coupling matrix of each network is defined as follows:

(a) Star-coupled networks:

$$c_{ij} = \begin{bmatrix} -N+1 & 1 & 1 & \cdots & 1 \\ 1 & -1 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & -1 \end{bmatrix}.$$

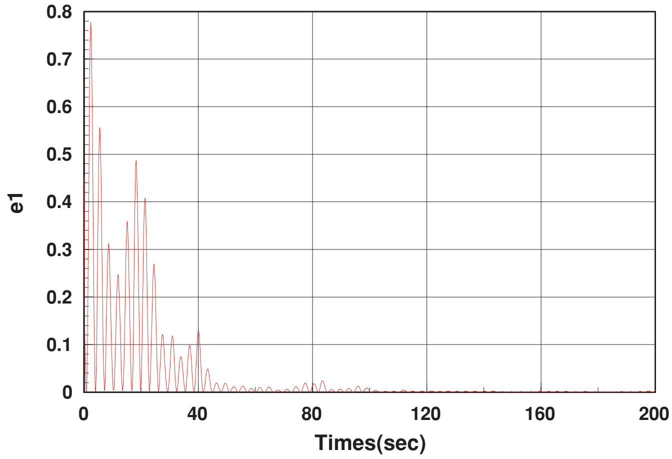

Figure 5. Synchronization displacement error in a nearest-neighbor coupled network ($K = 6$). The time step is $\delta t = 0.06$ and the initial conditions are $\left\{ \begin{smallmatrix} x_{i1}(0) \\ x_{i2}(0) \end{smallmatrix} \right\} = \left\{ \begin{smallmatrix} 1+2\times(i-N/2)/N \\ 1-2\times(i-N/2)/N \end{smallmatrix} \right\}$, with $d_1 = 0.001$ and $d_2 = 0$.

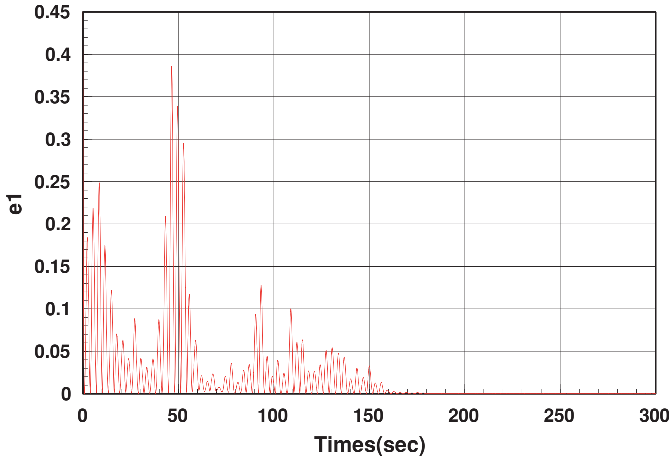

Figure 6. Synchronization displacement error in a nearest-neighbor coupled network ($K = 6$). The time step is $\delta t = 0.06$ and the initial conditions are $\left\{ \begin{smallmatrix} x_{i1}(0) \\ x_{i2}(0) \end{smallmatrix} \right\} = \left\{ \begin{smallmatrix} 1+2\times(i-N/2)/N \\ 1-2\times(i-N/2)/N \end{smallmatrix} \right\}$, with $d_1 = 0.001$ and $d_2 = 0$.

(b) Randomly coupled networks:

$$p(c_{ij} = 1) = r, \qquad p(c_{ij} = 0) = 1 - r, \qquad c_{ij} = c_{ji}, \qquad c_{ii} = -\sum_{\substack{j=1 \\ j\neq i}}^{N} c_{ij},$$

where $p(c_{ij} = 1)$ is the probability of $c_{ij} = 1$.

(c) Nearest-neighbor coupled networks:
Every node connects $K/2$ left-neighboring nodes and $K/2$ right-neighboring nodes, where K is an even number.

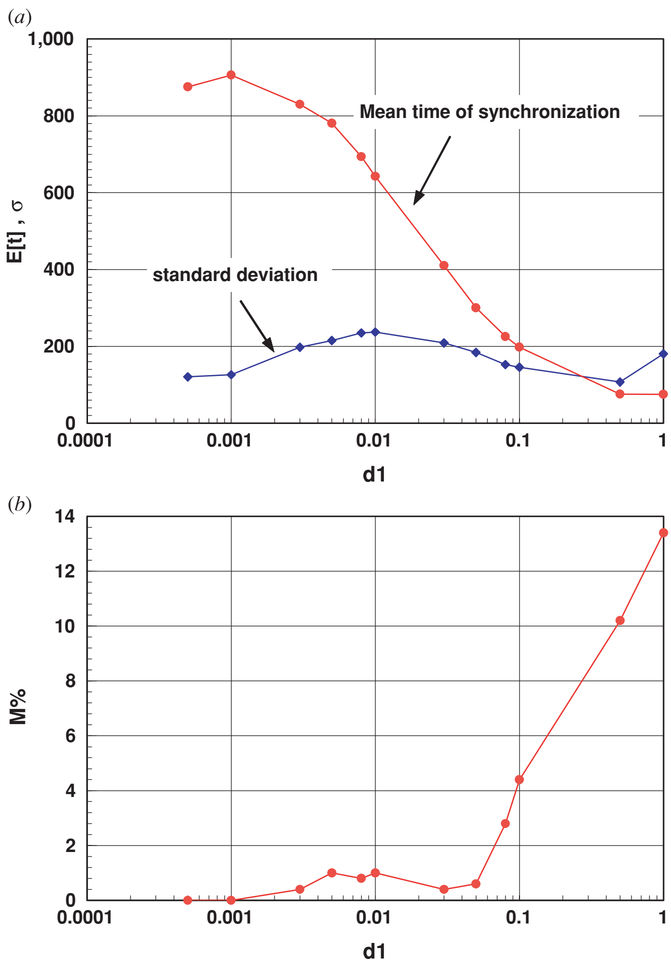

Figure 7. Mean time E[t] and standard deviation σ of synchronization and the proportion of failure to achieve synchronization, M, with different noise intensities, 500 samples were used for determining the system parameters. (*a*) Mean time and standard deviation of synchronization; (*b*) proportion of failure to achieve synchronization. Initial conditions are $\left\{ {x_{i1}(0) \atop x_{i2}(0)} \right\} = \left\{ {1+(i-N/2)/N \atop 1-(i-N/2)/N} \right\}$. Gaussian white noise intensity d_1 is 0.0005, 0.001, 0.003, 0.005, 0.008, 0.01, 0.03, 0.05, 0.08, 0.1, 0.5 and 1.0.

(d) Small-world networks:

To start, the initial matrix is the nearest-neighbor coupled network. Then, the elements change from $c_{ij} = 0$ to $c_{ij} = 1$ with the probability r. Finally, $c_{ii} = -\sum_{j=1, j\neq i}^{N} c_{ij}$ [26].

(e) Scale-free network:

The coupling matrix is defined as the same as that in [27].

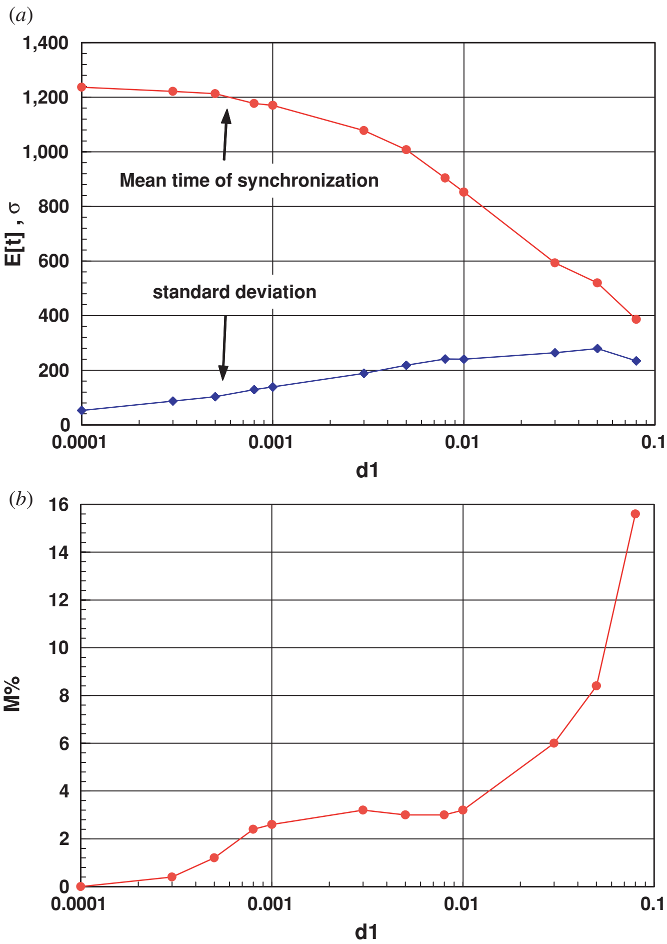

Figure 8. Mean time $E[t]$ and standard deviation σ of synchronization and the proportion of failure to achieve synchronization, M, with different noise intensities, 500 samples were used for determining the system parameters. (*a*) Mean time and standard deviation of synchronization; (*b*) proportion of failure to achieve synchronization. Initial conditions are randomly uniformly distributed in a semicircle, namely, $\left\{ {x_{i1}(0) \atop x_{i2}(0)} \right\} = \left\{ {1.10\cos((i-1)\times 3.14/N) \atop 1.10\sin((i-1)\times 3.14/N)} \right\}$. Gaussian white noise intensity d_1 is 0.0001, 0.0003, 0.0005, 0.0008, 0.001, 0.003, 0.005, 0.008, 0.01, 0.03, 0.05 and 0.08.

3. Numerical examples

The fourth-order Runge–Kutta method is used to simulate all the five types of dynamical networks. In the simulations, the network parameters are given as follows: $N = 500$, $a_{11} = a_{22} = 1, \beta = 1, \alpha_1 = -0.05, \alpha_2 = 0.05, \omega = 1.0$.

To measure the synchronization performance, define the errors by

$$e(t) = \begin{Bmatrix} e_1(t) \\ e_2(t) \end{Bmatrix} = \begin{Bmatrix} \dfrac{1}{N}\displaystyle\sum_{i=2}^{N} (x_{i1}(t) - x_{11}(t))^2 \\ \dfrac{1}{N}\displaystyle\sum_{i=2}^{N} (x_{i2}(t) - x_{12}(t))^2 \end{Bmatrix}. \tag{3}$$

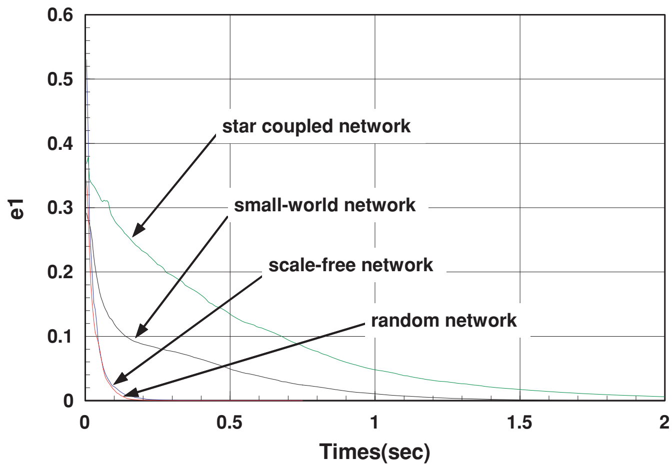

Figure 9. Synchronization displacement error in a star-coupled network, randomly coupled network ($r = 0.05$), small-world network ($K = 6$, $r = 0.005$), scale-free network (the initial network is a randomly coupled network ($r = 0.1$, $N_0 = 20$, and every new node brings $m_0 = 10$ connections to the network). The time step is $\delta t = 0.005$ and the initial conditions are randomly uniformly distributed, with $d_1 = 0.001$ and $d_2 = 0$.

As usual, the network achieves synchronization if $e(t)$ approaches zero as time tends to infinity.

First, the synchronization of networks subject only to parametric excitation, i.e., with $d_2 = 0$, is investigated. It can be seen from figures 1 and 2 that complete synchronization between the 500 nodes, with initial conditions $(x_{i1}(0), x_{i2}(0))^T$ randomly uniformly distributed in $(-1, 1) \times (-1, 1)$, can be achieved. It can also be seen that synchronization performances in different types of network are not quite the same. Obviously, synchronization in the nearest-neighbor coupled network is realized more difficultly than the other four types of networks.

The effect of initial conditions is also studied. Comparing figures 2 with 3, it can be observed that the synchronizing processes with different initial conditions in the same network are also different. It can be seen from figures 1–3, for instance, that network synchronization not only is influenced by the type of network, but also is sensitive to its initial conditions.

Furthermore, the effect of the intensity of the Gaussian white noise on the synchronization performance is studied. Herein, the nearest-neighbor coupled network with $K = 6$ is considered. It can be seen from figures 3–6 that a proper intensity of the Gaussian white noise could shorten the time of synchronization and the parametric excitation could enhance the network synchronization to a certain extent. It should be pointed out that the results for the error $e_2(t)$ are similar to those shown in figures 1–6. In order to make this observed phenomenon more creditable, multi-samples tests are carried out. To do so, several new parameters are first introduced.

(1) Time of synchronization, $t_i (i = 1, 2, \ldots, m)$, where $m = 500$ is the sample number: when $\sum_{j=k}^{k+150} e_{1j} \leqslant (150/500)$ and $\sum_{j=k}^{k+150} e_{2j} \leqslant (150/500)$, it is considered to have achieved synchronization, and define $t_i = \delta t \times (k + 75)$, where δt and k denote the time step and the step number, respectively. If $t \geqslant 1500$, it is considered failing to achieve synchronization.
(2) Mean time of synchronization, $E[t]$. It represents the ability of network to achieve synchronization.

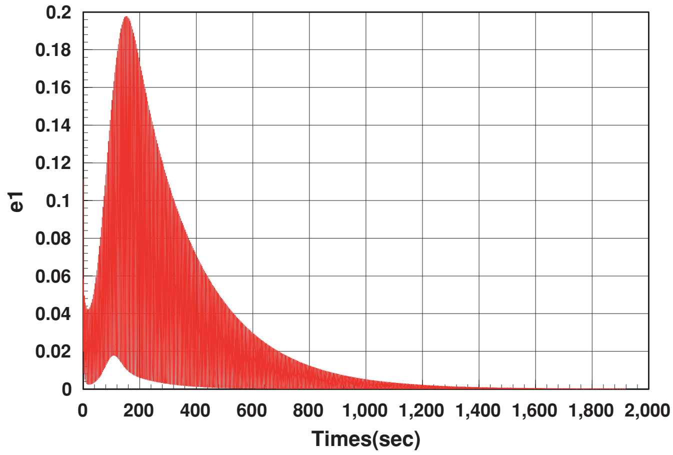

Figure 10. Synchronization displacement error in a nearest-neighbor coupled network ($K = 6$). The time step is $\delta t = 0.06$ and the initial conditions are randomly uniformly distributed, with $d_1 = 0.001$ and $d_2 = 0$.

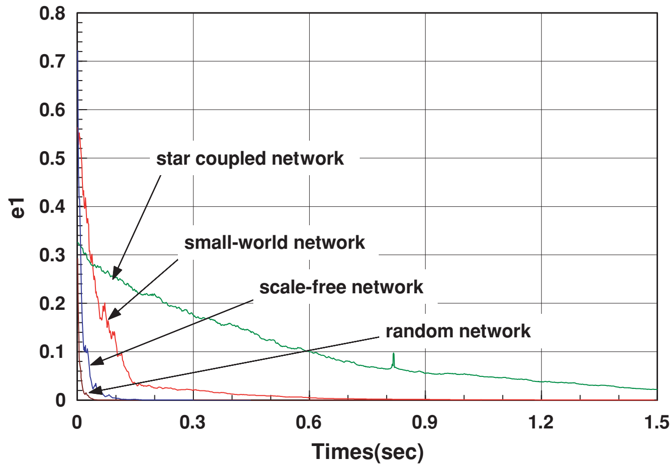

Figure 11. Synchronization displacement error in a star-coupled network, randomly coupled network ($r = 0.05$), small-world network ($K = 6$, $r = 0.005$), scale-free network (the initial network is a randomly coupled network ($r = 0.1$, $N_0 = 20$, and every new node brings $m_0 = 10$ connections to the network). The time step is $\delta t = 0.001$ and the initial conditions are randomly uniformly distributed, with $d_1 = 0.001$ and $d_2 = 0$.

(3) Standard deviation σ: it represents the degree of stability of synchronization.
(4) Proportion of failure to achieve synchronization, M: it represents the ability of achieving desynchronization due to parametric excitation by the Gaussian white noise.

It can be shown from figures 7 and 8 that the time of synchronization falls down with the increase in the noise intensity within a certain range. In other words, a proper Gaussian white noise can enhance the ability of network synchronization. When the noise intensity is greater than a certain value, the time of synchronization climbs up again. Note that the proportion of failure to achieve synchronization increases as well. Especially in figure 8(b), M increases rapidly when $d_1 > 0.01$. Therefore, the choice of noise intensity is not unlimited. Several numerical simulations with different initial conditions show that accelerating synchronization subject to parametric excitation does not always happen. For example, when the initial conditions are randomly uniformly distributed in $(-1, 1) \times (-1, 1)$, the ability of enhancing network synchronization is not so visible and even desynchronization occurs due to parametric excitation in many occasions.

The case where the network has random coupling only, i.e., with $d_1 = 0$, has also been simulated and analyzed. Figures 9 and 10 show the synchronization performances in all the five types of complex networks, all with $d_2 = 0.001$, while figure 11 shows the synchronization in only four types of networks, all with $d_2 = 0.01$. It can be seen from figure 11 that the ways to evolve to synchrony in the four networks are more fluctuant than the one shown in figure 9.

4. Conclusions

In this paper, the important yet difficult problem of complex network synchronization subject to parametric excitation of Gaussian white noise has been investigated numerically. It is found that unidirectionally coupled dynamical networks within proper parametric excitation can achieve synchronization. It is also found that the noise perturbation can even accelerate network synchronizability in some cases. Synchronization of randomly coupled networks has also been studied, revealing that randomly coupled networks can also achieve synchronization subject to proper noise perturbations. In the future, mathematical analysis of the stochastic networks need to be carried out, so as to provide a theoretical background for supporting the numerical simulations and to provide in-depth explanation of the experimental observations.

Acknowledgments

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